Unconventional odd-parity superconductivity in the ladder compounds

Mahito Kohmoto and Masatoshi Sato
Institute for Solid State Physics, University of Tokyo, 7-22-1 Roppongi, Minato-ku, Tokyo, Japan

Superconductive properties of the two-leg ladder compounds are studied theoretically. The antiferromagnetic fluctuations are considered because of the good nesting of the Fermi surfaces. The attractive interaction which is most likely due to the electron-phonon coupling is also taken into account. Under this circumstance, it is shown that the superconductivity has two sets of spin-triplet pairings. The gaps for both pairings have no node.

A number of unconventional superconductivities are under intensive study in recent years. Examples include the high T_c cuprates, heavy fermions, Sr_2RuO_4, organic conductors etc. The common property of these compounds is the existence of antiferromagnetic(AF) fluctuations. Because of this a large number of works are devoted to understand the unconventional nature from AF fluctuations. As a result most of them neglect the electron-phonon interactions which explain most of the superconductors [1] before the discovery of the high T_c cuprates. Although, there exist a few attempts to explain the unconventional nature of superconductivity from attractive interactions (which most likely come from electron-phonon coupling) under the influence of AF fluctuations [2–5].

Recently the ladder compounds which may have some connections with the high T_c cuprates were found. The crystal structures are different but the basic role of Cu and O looks similar. Hiroi et al. [6] were the first to synthesize the family of layer compounds $Sr_{n-1}Cu_nO_{2n-1}$, which have arrays of parallel line defects. Nearly ideal ladder compounds should result. The first member ($n = 2$ or $SrCu_2O_3$) has two-leg ladders, the second ($n = 3$ or $Sr_2Cu_3O_5$) has three-leg ladders and so on. It was also clearly shown that the material $LaCuO_{2.5}$ has an insulator-metal transition upon hole doping by substitution of Sr^{2+} for La^{3+} but no sign of superconductivity was observed [7]. There have been considerable interests in magnetism of the ladder compounds [8].

The ladder material $(Sr, Ca)_{14}Cu_{24}O_{41-\delta}$ was synthesized by MaCarron et al. [9] and by Siegrist et al. [10]. In this compound superconductivity was found by Uehara et al. [11]. It appears at around $T_c \sim 10$K but only under high pressure more than 3GPa. Due to this limitation essential properties of these compounds have not yet clarified by experiments. In particular the nature of superconductivity is not well understood.

In this paper we study the superconductive properties of two-leg(two-chain) ladders theoretically. In order to obtain universal features we only consider the minimum model which is supposed to give essential features of superconductivity. Details which are pertinent to specific compounds are not considered. Neither did we try to estimate the values of physical quantities like T_c. This is because, for quantitative predictions, one needs to fix various physical quantities which are not known theoretically or experimentally. Although, as shown below, T_c is similar to those of the classical BCS superconductors.

The existence of unconventional spin-triplet superconductivity in the ladder systems is shown. This result seems to be robust and there is a possibility to find unconventional superconductivity in other ladder materials which include more than two legs ladders.

–Band structure

The energy dispersion of the noninteracting two-chain ladder is given

$$\varepsilon(k) = -2t \cos k_x \pm t', \tag{1}$$

where t is the transfer integral in the x-direction which is along the chains (usually denoted as c-axis direction), and t' is the transfer integral between the two chains. The two transfer integrals, t and t' have the same order of magnitude. The sign in front of t' represents parity with respect to exchange of the two chains.

The inter-ladder hoppings are small and estimated to be $t'' \sim 1/20$ to $1/30\ t'$, and we shall neglect them hereafter.

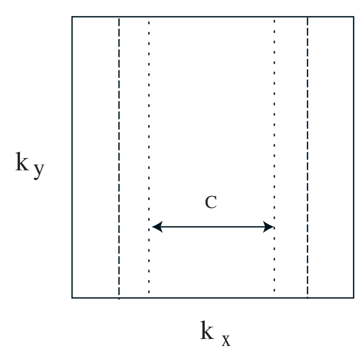

FIG. 1. The Fermi surfaces in the first Brillouin zone. The broken lines are the Fermi surfaces of the even-parity (with respect to exchange of the two chains) lower band and the dotted lines are the Fermi surfaces of the odd-parity upper band.

There are four Fermi surfaces as shown in Fig. 1. The superconductive pairs with finite momenta are in general have smaller binding energies compared with those with momentum zero, *i.e.* those consist of electrons in the same band (either + or − in Eq.(1)). Therefore we shall consider only one band and the two Fermi surfaces for the moment.

–Attractive interaction
First let us study interactions which are attractive and do not change spins. See Fig. 2. The general form is

$$H_{\rm int} = -\sum_{k,k'}\sum_{\alpha,\beta} : f(q) a^\dagger_{k+q,\alpha} a_{k,\alpha} a^\dagger_{k'-q,\beta} a_{k',\beta} :, \qquad (2)$$

where $f(q) > 0$ represents an attractive interaction and mainly depends on q_x. It has a peak at $q_x = 0$. Here $::$ denotes creation-annihilation normal ordering. This interaction most likely comes from electron-phonon coupling. Let us concentrate on the interactions which are relevant to $(k, -k)$ pairing. Thus choose $k' = -k$ and put $q = k'' - k$

$$H_{\rm int} = \sum_{k,k''}\sum_{\alpha,\beta} f(k''-k) a^\dagger_{k'',\alpha} a^\dagger_{-k'',\beta} a_{k,\alpha} a_{-k,\beta}. \qquad (3)$$

Let us introduce $V_{s_1 s_2 s_3 s_4}(k, k')$ by

$$H_{\rm int} = \frac{1}{2}\sum_{k,k'}\sum_{s_1,s_2,s_3,s_4} V_{s_1 s_2 s_3 s_4}(k,k') a^\dagger_{-k,s_1} a^\dagger_{k,s_2} a_{k',s_3} a_{-k',s_4}, \quad (4)$$

where $s_i's$ are spin indices. By the symmetry

$$\begin{aligned} V_{s_1 s_2 s_3 s_4}(k,k') &= -V_{s_2 s_1 s_3 s_4}(-k,k') \\ &= -V_{s_1 s_2 s_4 s_3}(k,-k'), \end{aligned} \qquad (5)$$

we obtain

$$\begin{aligned} &V_{s_1 s_2 s_3 s_4}(k,k') \\ &= \frac{1}{2}\{f(k+k') + f(-k-k'))\}\delta_{s_1 s_3}\delta_{s_2 s_4} \\ &-\frac{1}{2}\{f(k-k') + f(-k+k')\}\delta_{s_1 s_4}\delta_{s_2 s_3}. \end{aligned} \qquad (6)$$

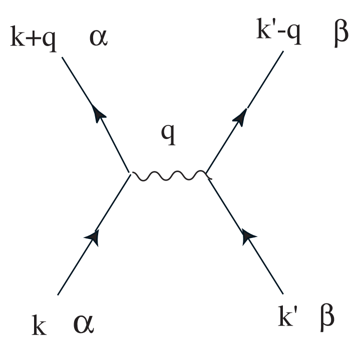

FIG. 2. Interaction which does not involve spin. The spins does not change by the interaction.

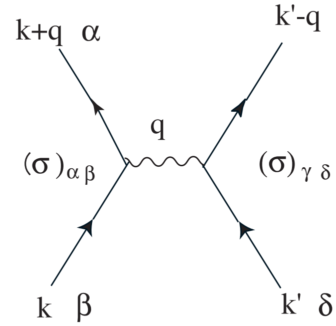

FIG. 3. Interaction which involves spins. The spins can changed by the interaction.

–Magnetic interaction
The general form of magnetic interaction is written (see Fig. 3)

$$\begin{aligned} &H^{\rm m}_{\rm int} \\ &= -\sum_{k,k',q}\sum_{\alpha,\beta,\gamma,\delta} : J_x(q)(\sigma_x)_{\alpha\beta}\cdot(\sigma_x)_{\gamma\delta} a^\dagger_{k+q,\alpha} a_{k,\beta} a^\dagger_{k'-q,\gamma} a_{k',\delta} : \\ &-\sum_{k,k',q}\sum_{\alpha,\beta,\gamma,\delta} : J_y(q)(\sigma_y)_{\alpha\beta}\cdot(\sigma_y)_{\gamma\delta} a^\dagger_{k+q,\alpha} a_{k,\beta} a^\dagger_{k'-q,\gamma} a_{k',\delta} : \\ &-\sum_{k,k',q}\sum_{\alpha,\beta,\gamma,\delta} : J_z(q)(\sigma_z)_{\alpha\beta}\cdot(\sigma_z)_{\gamma\delta} a^\dagger_{k+q,\alpha} a_{k,\beta} a^\dagger_{k'-q,\gamma} a_{k',\delta} : . \end{aligned} \qquad (7)$$

In general $J_i(q)(>0)$'s $(i = x, y, z)$ are different each other, but for simplicity we treat explicitly only the case $J_x(q) = J_y(q) = J_z(q) \equiv J(q)$ in the following. The anisotropy of the magnetic interaction amplifies the tendency toward the odd-parity superconductivity, and it determines the direction of the $\vec{d}$ vector (which is defined by Eq.(16)) [5]. If the anisotropy is strong enough, the odd-parity superconductivity is realized even if $f(q) = 0$.

By considering only the terms which contribute $(k, -k)$ pairings, we have (by putting $k = -k'$)

$$\begin{aligned} &H^{\rm m}_{\rm int} \\ &= -\sum_{k,q}\sum_{\alpha,\beta,\gamma,\delta} : J(q)(\vec{\sigma})_{\alpha\beta}\cdot(\vec{\sigma})_{\gamma\delta} a^\dagger_{k+q,\alpha} a_{k,\beta} a^\dagger_{-k-q,\gamma} a_{-k,\delta} : . \end{aligned} \qquad (8)$$

Put $q = k'' - k$ then

$$\begin{aligned} &H^{\rm m}_{\rm int} \\ &= -\sum_{k,k''}\sum_{\alpha,\beta,\gamma,\delta} : J(k''-k)(\vec{\sigma})_{\alpha\beta}\cdot(\vec{\sigma})_{\gamma\delta} a^\dagger_{k'',\alpha} a_{k,\beta} a^\dagger_{-k'',\gamma} a_{-k,\delta} : \\ &= \sum_{k,k''}\sum_{\alpha,\beta,\gamma,\delta} J(k''-k)(\vec{\sigma})_{\alpha\beta}\cdot(\vec{\sigma})_{\gamma\delta} a^\dagger_{k'',\alpha} a^\dagger_{-k'',\gamma} a_{k,\beta} a_{-k,\delta}. \end{aligned} \qquad (9)$$

By comparing this equation (9) and Eq.(4), and using the symmetry (5), the magnetic interaction is written

$$V^{\rm m}_{s_1s_2s_3s_4}(k,k')$$
$$= \frac{1}{2}\{J(k+k')+J(-k-k')\}(\vec{\sigma})_{s_1s_3}\cdot(\vec{\sigma})_{s_2s_4}$$
$$-\frac{1}{2}\{J(k-k')+J(-k+k')\}(\vec{\sigma})_{s_1s_4}\cdot(\vec{\sigma})_{s_2s_3}. \quad (10)$$

–Gap equation [1]
We use the weak coupling BCS theory. It is possible that the strong-coupling corrections modify the results quantitatively but not qualitatively because $T_{\rm c}$ is not so high. The gap equation is

$$\Delta_{ss'}(k) = -\sum_{k',s_3,s_4} V_{s'ss_3s_4}(k,k') \times \frac{\Delta_{s_3s_4}(k')}{2E_{k'}}\tanh\left(\frac{\beta E_{k'}}{2}\right), \quad (11)$$

where the (scalar) excitation energy $E_{k'}$ is given by

$$E_k = \sqrt{\varepsilon(k)^2 + \frac{1}{2}{\rm tr}(\Delta(k)\Delta(k)^\dagger)}, \quad (12)$$

For even-parity states (spin-singlet pairing) one can write [12]

$$\Delta(k) = i\sigma_y\psi(k) \quad (13)$$

with $\psi(k) = \psi(-k)$. Thus the gap equation for even-parity states is

$$\psi(k) = -\sum_{k'} V^{\rm even}_{kk'}\frac{\psi(k')}{2E_{k'}}\tanh\left(\frac{\beta E_{k'}}{2}\right). \quad (14)$$

where $V^{\rm even}_{kk'}$ is obtained from Eq.(6) and Eq.(10) as

$$V^{\rm even}_{kk'}$$
$$= -\frac{1}{2}\{f(k+k')+f(-k-k')+f(k-k')+f(-k+k')\}$$
$$+\frac{3}{2}\{J(k+k')+J(-k-k')+J(k-k')+J(-k+k')\}. \quad (15)$$

For odd-party states (spin-triplet pairing) one can write [12]

$$\Delta(k) = i(\vec{d}(k)\cdot\vec{\sigma})\sigma_y \quad (16)$$

with $\vec{d}(k) = -\vec{d}(-k)$. The gap equation for odd-parity states is

$$\vec{d}(k) = -\sum_{k'} V^{\rm odd}_{kk'}\frac{\vec{d}(k')}{2E_{k'}}\tanh\left(\frac{\beta E_{k'}}{2}\right). \quad (17)$$

where $V^{\rm odd}_{kk'}$ is obtained from Eq.(6) and Eq.(10) as

$$V^{\rm odd}_{kk'}$$
$$= \frac{1}{2}\{f(k+k')+f(-k-k')-f(k-k')-f(-k+k')\}$$
$$+\frac{1}{2}\{J(k+k')+J(-k-k')-J(k-k')-J(-k+k')\}. \quad (18)$$

–BCS approximation [1]
Following the BCS treatment of superconductivity we make further approximations. Namely the pairing interactions are constant within the cutoff and zero outside the cutoff. The function $f(q)$ has a peak at $q_x = 0$ but it is approximated by a constant $V > 0$. Therefore this interaction corresponds to the phonon-mediated interaction in the classical BCS theory. In addition to this interaction the ladder compounds have interactions due to AF fluctuations which are induced by nesting of the Fermi surfaces in a similar fashion to some of the organic conductors or the high $T_{\rm c}$ cuprates in the low doping region. Since the Fermi surfaces are straight and parallel, the nesting is perfect. Therefore the nesting vector is $Q = (c, k_y, k_z)$ where c is the distance between the two Fermi surfaces of the same band. See Fig.1. (Nesting is also perfect between the Fermi surfaces of different bands. But these are not relevant to the superconducting pairing of $(k,-k)$.) Therefore there exist AF fluctuations with nesting vector Q. Now we take the magnetic interaction $J(q)$ to be constant $J(Q) > 0$ when q is is close to Q and connects two points in different Fermi surfaces (of the same band) within the cutoff. Then Eq.(15) gives

$$V^{\rm even} = -V + 3J(Q) \quad (19)$$

for even-parity states, and Eq.(18) gives

$$V^{\rm odd} = -V + J(Q) \quad (20)$$

for odd-parity states. Therefore we always have $-V^{\rm odd} > -V^{\rm even}$ as long as magnetic interactions are present. It leads to odd-parity (spin-triplet) pair condensation with

$$T_{\rm c} = 1.13\omega_{\rm D}e^{1/\{N(k_{\rm F})V^{\rm odd}\}}. \quad (21)$$

where $\omega_{\rm D}$ is a cutoff.

When the anisotropy of AF fluctuations is taken into account, $V^{\rm odd}$ is replaced by

$$V^{\rm odd} = \begin{cases} -V - J_x(Q) + J_y(Q) + J_z(Q) & \text{for } d_x \neq 0 \\ -V - J_y(Q) + J_z(Q) + J_x(Q) & \text{for } d_y \neq 0 \\ -V - J_z(Q) + J_x(Q) + J_y(Q) & \text{for } d_z \neq 0 \end{cases}. \quad (22)$$

It can be seen from Eq.(22) that if AF fluctuations perpendicular to the $\vec{d}$ vector are strong enough, there is no superconductive condensation. It also can be seen that AF fluctuations parallel to the $\vec{d}$ vector promote the odd-parity superconductivity. It is rather straightforward to show that there is no node in the gaps from the gap equation (11).

–Discussions
To conclude, we study the unconventional superconductive nature of the two-leg ladder compounds. The attractive interactions which are most likely due to the electron-phonon coupling are considered. AF fluctuations from the good nestings of the Fermi surfaces are also included.

Our main conclusion is that the two-leg ladder compounds have two sets of spin-triplet superconductive states without nodes in the gap corresponding the two sets of the Fermi surfaces. These superconductivities are suppressed by AF fluctuations perpendicular to the $\vec{d}$ vector and vanish if the AF fluctuations are strong enough. Our results also apply to the ladder compound with more than two legs.

It is possible to have two $T_{\rm c}$'s corresponding to the two bands, respectively. However it is likely to have a single $T_{\rm c}$ due to some proximity effects induced by the interactions which are not included here.

Acknowledgments

It is a pleasure to thank J. Akimitsu and N. Mori for useful discussions.

[1] J. Bardeen, L.N. Cooper,and J.R. Schrieffer, Phys. Rev. **106**, 162; **108**, 1175 (1957).
[2] J. Friedel and M. Kohmoto, (preprint) cond-mat/9901065.
[3] I.Chang, J. Friedel and M. Kohmoto, (preprint) cond-mat/9908214.
[4] M. Kohmoto and M. Sato, (preprint) cond-mat/0001331.
[5] M. Sato and M. Kohmoto, (preprint) cond-mat/0003046.
[6] Z. Hiroi, M. Azuma, M. Takano, and Y. Bando, J. Solid State Chem. **95**, 230 (1991).
[7] Z. Hiroi and M. Takano, Nature **377**, 41 (1995).
[8] For a review, see *e.g.* E. Dagotto and T.M. Rice, Science **271**, 618 (1996).
[9] E.M. McCarron, M.A. Surbramanian, J.C. Calabrese, and R.L. Harlow, Mater. Res. Bull. **23**, 1355 (1988).
[10] T. Siegrist, L.F. Schneemeyer, S.A. Sunshine, J.V. Waszczah and R.S. Roth, Mater. Res. Bull. **23**, 1429 (1988).
[11] M. Uehara, T. Nagata, J. Akimitsu, H. Takahashi, N. Mori, and K. Kinoshita, J. Phys. Soc. Jpn. **65**, 2764 (1996).
[12] See *e.g.* A. Leggett, Rev. Mod. Phys. **47**, 331 (1975); M. Siegrist and K. Ueda, Rev. Mod. Phys. **63**, 239 (1991).